MATHEMATICS STD X

CHAPTER 1

REAL NUMBERS

NUMBER SYSTEMS

When we think about numbers, 1,2,3,4,5,6,7,8,9,………will come to our mind first, why ? It is because they are the numbers we used for counting from our nursery classes onwards and we called them counting numbers. Then we started to add and subtract these numbers. At one point when we subtract a number from itself, like 7 – 7 and 8 – 8 the answer is Zero. Then we gradually became aware the importance of 0 as a number. During subtraction a question like 5 – 7 leads us to negative numbers and then we became aware that there are positive numbers also.

For our convenience we arranged these numbers in different groups.

1,2,3,4,5,6,7,8,9,10. 11.12. …………………..

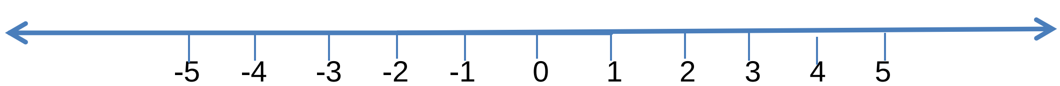
Counting numbers are the first group. They are called *Counting numbers or Natural numbers*. We denote this collection of numbers by the symbol **N.**

We include the number 0 in the Natural number group and formed another group and it is called *Whole numbers*.

0,1,2,3,4,5,6,7,8,9,10,11,12, …………………

We denote this collection of numbers by the symbol **W.**

To get a clear view of numbers we arranged them on a line and we call it number line. On a number line we arranged 0 in the middle and negative numbers and positive numbers on either sides of zero.



The collection of all negative numbers , Zero and positive numbers is called *Integers.*

…………..-5,-4,-3,-2,-1,0,1,2,3,4,5,……………………

We denote this collection of numbers by the symbol **Z.**

We got two more mathematical operations, multiplication and division. During division we came across the numbers like ½ , ¾ etc. they are called fractions. We include fractions in the group of integers and formed a new group.

………-5,-4,-3,-2,-1,- ¾, -½, 0, ½ , ¾, 1,2,3,4,5……….

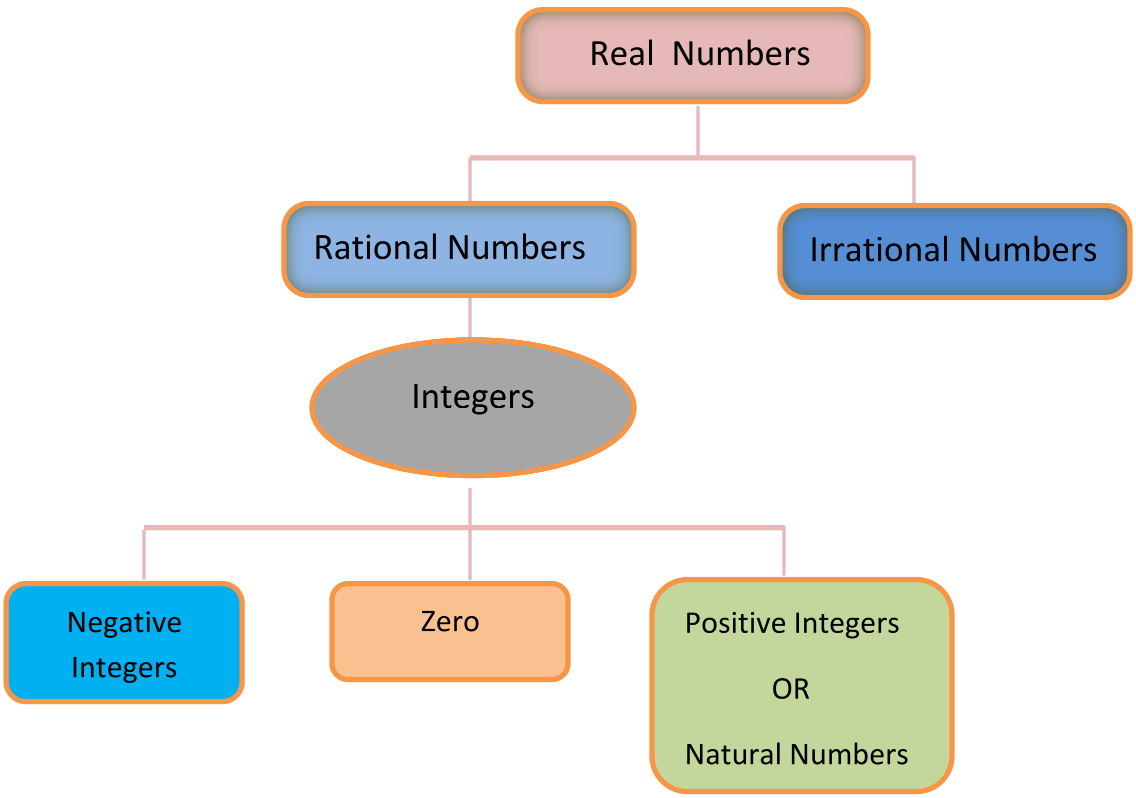
This group is called the *Rational numbers.* This collection is denoted by the symbol **Q.** In this group we got integers and fractions. Can we write the integers in the form of fractions ? Yes, the integer 2, can be written as $\frac{10}{5}$ or $\frac{2}{1}$. In the similar way all the integers can be written in the form of a fraction.

So we say a *Rational number* can be written in the form $\frac{p}{q}$, where *p* and *q* are integers and q ≠ 0.

There are numbers in the number system that cannot be written in the form of $\frac{p}{q}$ but they can be marked on the number line. They are $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, π, 0.1011011101110…… those numbers are called *Irrational numbers*. The collection of numbers that are rational and irrational is called *Real numbers*.

REAL NUMBERS.

The Structure of Real Number System



In this chapter we begin with two important properties of positive integers.

Euclid's division algorithm : Any positive integer a can be divided by another positive integer b in such a way that it leaves a quotient q and remainder r that is smaller than b.

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Fundamental Theorem of Arithmetic : Every composite number can be expressed as a product of primes in a unique way.

According to Euclid's division algorithm if we divide a positive integer, let $a = 17$ by another positive integer, Let $b = 5$ then the quotient $q = 3$ and the remainder $r = 2$. Which is less than 5.

But we know that $Dividend = Divisor \times Quotient + Reminder$.

That is $17 = 5 \times 3 + 2$

or $a = b * q + r$ *(This is Euclid's division lemma)*

Euclid's Division Lemma : Given positive integers a and b there exist unique integers q and r satisfying $a = bq + r$ where $0 \leq r < b$

We use the above theorem to find the HCF of two positive integers.

Let us first go back and see what is HCF or Highest Common Factor.?

Let us consider the numbers 130 and 50

The factors of 130 = 2 x 5 x 13

The factors of 50= 2 x 5 x 5

The common factors of 130 and 50 are 2, 5, and 10

2	130
5	65
	13

But the Highest of them is 10. So HCF is 10. In other words HCF is 5 x 2 = 10, ie

HCF is the product of the smallest power of each common prime factor in the numbers

Now let us see how Euclid's division lemma can be used to find out HCF of 130 and 50.

According to lemma if a=130 and b=50 there will be q and r satisfying a = b * q + r

To find q and r we can divide 130 by 50 then q = 2 and r = 30

That is 130 = 50 x 2 + 30

Now apply again the division lemma by taking the divisor 50 as a and the remainder 30 as b.

Dividing 50 by 30 we get q = 1 and r = 20

That is 50 = 30 x 1 + 20

Apply the lemma again by using 30 and 20

Divide 30 by 20 we get 30 = 20 x 1 + 10

Divide 20 by 10 we get 20 = 10 x 2 + 0 Here the remainder is zero, so the divisor at this stage will be the required HCF, that is 10

LCM is the product of the greatest power of each prime factor involved in the number.

LCM of 130 and 50

The factors of 130 = 2 x 5 x 13

The factors of 50= 2 x 5 x 5 = 2×5^2

LCM = $2 \times 5^2 \times 13 = 6500$